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ABSTRACTS

Noncommutative algebraic dynamics: Ergodic transformations of pro-2-groups

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Let $\mathcal{G} = \langle G, \Omega \rangle$ be a group G with a set of operators Ω; let $w(x)$ be a *polynomial* over $\mathcal{G}$; that is

$$w(x) = g_1 x^{n_1\omega_1} g_2 x^{n_2\omega_2} \dots g_k x^{n_k\omega_k} g_k,$$

where $n_i \in \mathbb{Z}$, $\omega_i \in \Omega$, $g_i \in G$. (Note: we represent the group operation in a multiplicative form; that is, $(uv)^\omega = u^\omega v^\omega$, $u^{n\omega} = (u^n)^\omega = (u^\omega)^n$ for $u, v \in G$, $\omega \in \Omega$, and a rational integer $n \in \mathbb{Z}$).

Problem: under what conditions the transformation

$$a \mapsto w(a) \quad (a \in G)$$

is ergodic?

The problem seems to be infeasible whenever the group G is non-solvable. For a solvable group the problem could be handled.

A counterpart of the problem for finite commutative rings leads to the ergodic theory on the space of p-adic integers $\mathbb{Z}_p$, the latter being a projective limit of residue rings modulo p^n, $n = 1, 2, \dots$. In case G is a finite solvable group, the problem stated above leads to the ergodic theory on a pro-2-group D_∞, which is a non-Archimedean metric space, a projective limit of finite dihedral groups of order 2^n.

Comparing to the ergodic theory on $\mathbb{Z}_p$, which machinery is based on the p-adic differential calculus, the techniques of the developed theory is based on a modified version of the free differential calculus originally developed by R. Fox to study some problems of knot theory.

The problem is motivated by some cryptographic issues; the resulting theory may have applications to computer science and cryptography.

Consciousness, spacetime and quantum mechanics of macroscopic systems

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We propose the quantum mechanical description of complex systems should be performed using two types of causality relation: the ordering relation ($x \prec y$) and the subset relation ($A \subset B$). The structures with two ordering operations, called the *causal sites*, have been already proposed in context of quantum gravity (Christensen and Crane, 2005). We suggest they are also common to biological physics and may describe how the brain works. In the spirit of the Penrose ideas we identify the geometry of the space-time with universal field of consciousness. The latter has its evident counterparts in ancient Indian philosophy and provides a framework for unification of physical and mental phenomena.

p-adic Cosmology

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A great discovery of the last few years is the discovery of the current accelerated expansion of the Universe. This acceleration seems to be explicable by the presence of a new component of matter called dark energy. To explain the dark energy and the Big Bang very exotic field models have been used in last years. String theories provide some of these models. Also many attempts has been made to building and testing stringy models of inflation.

p-Adic cosmology is related with p-adic strings. More realistic superstring nonlocal cosmological models are deformations of the p-adic cosmological model. In these models the Friedmann equations give a system of nonlinear nonlocal equations. Recent results of study of these equations will be presented.

New results in the applications of p–adic pseudodifferential equations to the protein dynamics

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On the First International conference on p-adic mathematical physics, we pointed out that the protein dynamics can be described by the p-adic pseudo-differential equation of ultrametric diffusion. We showed an ultrametric model for the ligand rebinding kinetics of myoglobine and demonstrated good agreement with experimental data. Our new application of p-adic pseudo-differential equations to the protein dynamics is related to the phenomenon of spectral diffusion in globular proteins. Spectral diffusion is a peculiar random process propagated on a "frequency line", which is observed by measurements of the absorption frequency of a marker injected into the protein macromolecule. Two distinguish feature are inherent in spectral diffusion: anomalously slow widening of the spectral diffusion kernel and aging effect. We present a model of spectral diffusion in proteins based on ultrametric description of the protein dynamics and exhibit excellent agreement with experimental data in this case too. These results support an idea that proteins are macromolecular structures with ultrametric order.

Stochastic processes in Q_p associated with nonlinear PDEs

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In the Eucledian case there exists a connection between a class of nonlinear integro-differential equations and Markov processes with jumps similar to one that there exists between nonlinear parabolic equations and diffusion processes investigated by Dalecky and Belopolskaya. We study the similar connections between integro-differential equations for scalar and vector valued functions defined on Q_p and corresponding jump processes. At the same time we construct the analogue of the Feynmann-Kac formula and study both scalar and matrix-valued multiplicative functionals of jump Q_p-valued processes.

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space. For a subset $\Gamma \in K$ denote by $\nu_\alpha(t,\Gamma)$ a Poisson process with the parameter $t\pi_\alpha(\Gamma)$ $\nu\alpha(t,\Gamma)$ is a random non-negative countably additive measure on the Borel σ-algebra $\mathcal{B}(Q_p\backslash 0)$ such that $\nu_\alpha(t,\Gamma_1)$ and $\nu_\alpha(t,\Gamma_2)$ are independent if $\Gamma_1\cap\Gamma_2=\emptyset$. Below we assume that $E\nu_\alpha(t,\Gamma)=t\pi_\alpha(\Gamma)$ where π_α is the Lévy measure

$$\pi_\alpha(dz)=\frac{p^\alpha-1}{1-p^{-\alpha-1}}\|z\|_p^{-\alpha-1}dz.$$

Consider a stochastic differential equation

$$d\xi(t)=\int_{Q_p} f(\xi(t),u(t,\xi(t)),z)\nu_\alpha(dt,dz),\quad \xi(s)=x\in K \tag{1}$$

where $f(x,u,z)$ is a K-valued deterministic function and $0\le s\le t\le T$.

Condition **C 1**. For a fixed $u(t,x)\in R^d$ we assume that $f_u(t,x,z)=f(x,u(t,x),z)$ satisfies common conditions to ensure the existence and uniqueness of a solution of a stochastic differential equation of the form (1). One can find these conditions in the book (A. Kochubei, Pseudo-differential equations and stochastics over non-archimedian fields. Marcel Dekker Inc). Assume in addition that $u_0(x)$ is a bounded scalar function defined on Q_p.

Set

$$u(s,x)=E_{s,x}[exp^{\int_0^T c(\xi(\tau),u(\tau,\xi(\tau)))d\tau}u_0(\xi(T))]. \tag{2}$$

Theorem 1. *Assume* **C 1** *holds and both $f(x,u,z)\in Q_p$ and $c(x,u)\in R^d\times R^d$ are Lipschitz continuous in $x\in Q_p$ and $u\in R^d$ in corresponding norms. Besides assume that they are of polynomial growth in $u\in R^d$ and bounded in x. Then there exists a unique solution of (1) (2). In addition the function $u(s,x)$ defined by (2) satisfies the following Cauchy problem*

$$\frac{\partial u(s,x)}{\partial s}+c(x,u(s,x))u(s,x)+$$

$$\frac{p^\alpha-1}{1-p^{-\alpha-1}}\int_{Q_p}[u(s,x+f(x,u(s,x),z))-u(s,x)]\|z\|_p^{-\alpha-1}dz=0. \tag{3}$$

$$u(T,x)=u_0(x). \tag{4}$$

Remark. Note that if instead of $\eta(t)=exp^{\int_0^T c(u(\tau,\xi(\tau))d\tau}\in R^1$, $\eta(0)=1$ we consider a solution of a linear equation in $R^d\times R^d$

$$d\eta(t)=c(t,\xi(t))\eta(t)dt+\int_{Q_p}\gamma(\xi(t),u(t,\xi(t)),z)\eta(t)\nu_\alpha(dt,dz),\eta(s)=I.$$

We can extend the above result and construct a probabilistic representation to the following Cauchy problem

$$\frac{\partial v(s,x)}{\partial s} + c(x, v(s,x))v(s,x) +$$

$$\frac{p^\alpha - 1}{1 - p^{-\alpha-1}} \int_{Q_p} [\gamma(x, v(s,x), z)[v(s, x + f(x, v(s,x), z))]\|z\|_p^{-\alpha-1} dz +$$

$$\frac{p^\alpha - 1}{1 - p^{-\alpha-1}} \int_{Q_p} [v(s, x + f(x, v(s,x), z)) - v(s,x)]]\|z\|_p^{-\alpha-1} dz = 0,$$

$$v(T, x) = u_0(x)$$

in the form $V(s,x) = E_{s,x}[\eta^*(T) u_0(\xi(T))]$.

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[1] Ya. Belopolskaya, *Systems of quasilinear integro-differential equations and Markov processes associated with them*, Probabilistic distributions in infinite dimensional spaces. Nauk. Dumka, (1978) p. 5-21.

On the first return problem for the ultrametric diffusion process

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Задача о первом возвращении для процесса ультраметрической диффузии

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Для случайного процесса описывающегося уравнением ультраметрической диффузии с оператором Владимирова поставлены и решены задачи о первом возвращении (достижении) системы в начальное состояние (достижения некоторой области). В данной работе получено уравнени для плотности распределения случайной величины момента времени первого возвращения (достижения) и исследованы его свойства.

Mumford dendrograms and discrete p-adic symmetries

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In this talk, we present an effective encoding of dendrograms by embedding them into the Bruhat-Tits trees associated to p-adic number fields. As an application, we show how strings over a finite alphabet can be encoded in cyclotomic extensions of $\mathbb{Q}_p$ and discuss p-adic DNA encoding. The application leads to fast p-adic agglomerative hierarchic algorithms similar to the ones recently used e.g. by A. Khrennikov and others. From the viewpoint of p-adic geometry, to encode a dendrogram X in a p-adic field K means to fix a set S of K-rational punctures on the p-adic projective line $\mathbb{P}^1$. To $\mathbb{P}^1 \setminus S$ is associated in a natural way a subtree inside the Bruhat-Tits tree which recovers X, a method first used by F. Kato in 1999 in the classification of discrete subgroups of $\mathrm{PGL}_2(K)$.

Next, we show how the p-adic moduli space $\mathfrak{M}_{0,n}$ of $\mathbb{P}^1$ with n punctures can be applied to the study of time series of dendrograms and those symmetries arising from hyperbolic actions on $\mathbb{P}^1$. In this way, we can associate to certain classes of dynamical systems a Mumford curve, i.e. a p-adic algebraic curve with totally degenerate reduction modulo p.

In the end, we indicate some of our results in the study of general discrete actions on $\mathbb{P}^1$, and their relation to p-adic Hurwitz spaces.

The p-adic quantum plane algebras and quantum Weyl algebra
(with Fana Tangara)

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Let q be a principal unit of the ring of valuation of a complete valued field K, extension of the field of p-adic numbers. Generalizing Mahler basis, K. Conrad has constructed orthonormal basis, depending on q, of the space of continuous functions on the ring of p-adic integers with values in K. Attached to q there are two models of the quantum plane and a model of the quantum Weyl algebra, as algebras of bounded operators on the space of p-adic continuous functions. For q not a root of unit, interesting orthonormal (orthogonal) families of these algebras are exhibited and providing p-adic completion of quantum plane and quantum Weyl algebras.

Schrodinger operators on local fields: Self-adjointness and path integral representations for propagators

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We study matrix valued operators of Schrodinger type over a local field, and discuss questions related to their self-adjointness. Also, a path integral formula for such operators is obtained.

Towards p-Adic Genomics

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The present talk is devoted to some results in foundations of p-adic modelling in genomics. Genomics is the study of the genomes which are whole hereditary information of organisms. Considering nucleotides, codons, DNA and RNA sequences, amino acids, and proteins as information systems, we have formulated the corresponding p-adic formalisms for their investigations. Each of these systems has its characteristic prime number used for construction of the related information space. Our formalism of p-adic genomic information systems can be applied to various concrete cases.

In particular, it is shown that degeneration of the genetic code is a p-adic phenomenon. Degeneration of the (vertebral mitochondrial) genetic code has a natural description on the 5-adic space of 64 codons $\mathcal{C}_5(64) = \{n_0 + n_1\, 5 + n_2\, 5^2 : n_i = 1, 2, 3, 4\}$, where n_i are digits related to nucleotides as follows: C = 1, A = 2, T = U = 3, G = 4. The smallest 5-adic distance between codons joins them into 16 quadruplets, which under 2-adic distance decay into 32 doublets. p-Adically close codons are assigned to one of 20 amino acids, which are building blocks of proteins, or code termination of protein synthesis.

We have also put forward a hypothesis on evolution of the genetic code assuming that primitive code was based on single nucleotides and chronologically first four amino acids.

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Path integrals for quadratic Langrangians on real, p-adic, and adelic spaces

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Path integrals in ordinary, p-adic and adelic quantum mechanics are considered. The corresponding probability amplitudes $\mathcal{K}(x^{''}, t^{''}; x', t')$ for two-dimensional systems with quadratic Lagrangians are analytically evaluated and obtained expressions are generalized to any finite-dimensional spaces. These exact general formulas are presented in the form which is invariant under interchange of the number fields $\mathbb{R} \longleftrightarrow \mathbb{Q}_p$ and $\mathbb{Q}_p \longleftrightarrow \mathbb{Q}_{p'}$, $p \neq p'$. This invariance shows that adelic path integral is an essentially fundamental object in mathematical physics of quantum phenomena.

The Ultrametric Corona Problem

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Let A be the algebra of bounded analytic functions in the disk $|x| < 1$ of a complete algebraically closed field. If a maximal ideal is the kernel of a unique continuous multiplicative semi-norm, this semi-norm belongs to the closure of the set of continuous multiplicative semi-norms defined by points of D. If K is strongly valued, the set continuous multiplicative semi-norm defined by points of D is dense in the set of of continuous multiplicative semi-norms defined by maximal ideals. Various other results are showed.

On orthogonal wavelets and Walsh series

Yu. A. Farkov

Об ортогональных вейвлетах и рядах Уолша

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Основы теории рядов и преобразований Уолша как одного из разделов гармонического анализа изложены в монографиях [1, 2]. В то время как характерами группы вращений окружности являтся гармоники e^{ikt}, функции Уолша являются характерами канторовой диадической группы $\mathcal{C}$. Ортогональные вейвлеты и соответствующие им масштабирующие функции, представимые в виде лакунарных рядов Уолша, изучались в [3] - [6]. В частности, в работе [3] найдены условия, при которых ортогональные вейвлеты порождают безусловные базисы в пространствах $L^q(\mathcal{C})$, $1 < q < \infty$, а в статье [5] получены точные по порядку оценки модулей гладкости масштабирующих функций, построенных в [3, 4]. Как отмечено в [7], специфика построения вейвлетов на канторовой группе $\mathcal{C}$ связана с тем обстоятельством, что эта группа (как и аддитивная группа $\mathbb{Q}_p$ рациональных p-адических чисел) содержит открытую компактную подгруппу.

В докладе будет рассказано об основных свойствах ортогональных вейвлетов с компактными носителями на локально компактной абелевой группе G, являющейся слабым прямым произведением счетного множества циклических групп p-го порядка (группу G иногда называют группой Виленкина; при $p = 2$ эта группа изоморфна канторовой группе $\mathcal{C}$). Элементами группы G являютс последовательности вида

$$x = (x_j) = (\dots, 0, 0, x_k, x_{k+1}, x_{k+2}, \dots),$$

где $x_j \in \{0, 1, \dots, p-1\}$ для $j \in \mathbb{Z}$ и $x_j = 0$ для $j < k = k(x)$. Групповая операция $\oplus$ на G определяется как покоординатное сложение по модулю p, а топология вводится полной системой окрестностей нуля: $U_l = \{\, (x_j) \in G \;:\; x_j = 0 \;\text{ для }\; j \le l \,\}$, $l \in \mathbb{Z}$. Выделим в G дискретную подгруппу $H = \{(x_j) \in G \,|\, x_j = 0 \text{ для } j > 0\}$ и определим автоморфизм $A \in \operatorname{Aut} G$ по формуле $(Ax)_j = x_{j+1}$. Отображение $\lambda : G \to [0, +\infty)$ определим равенством

$$\lambda(x) = \sum_{j \in \mathbb{Z}} x_j p^{-j}, \qquad x = (x_j) \in G.$$

Образом подгруппы H при отображении λ является множество целых неотрицательных чисел: $\lambda(H) = \mathbb{Z}_+$. Для каждого $\alpha \in \mathbb{N}$ через $h_{[\alpha]}$ обозначим элемент из G такой, что $\lambda(h_{[\alpha]}) = \alpha$ и все компоненты последовательности $h_{[\alpha]}$, начиная с некоторого номера, равны 0. Положим также $h_{[\alpha]} = \theta$ для $\alpha = 0$. При любом целом $n \geq 2$ найдены необходимые и достаточные условия дл того, чтобы решения масштабирующих уравнений вида

$$\varphi(x) = \sum_{\alpha=0}^{p^n-1} a_\alpha \varphi(Ax \ominus h_{[\alpha]}) \tag{1}$$

генерировали кратномасштабные анализы в $L^2(G)$. Показано, что каждое такое решение φ разлагается в лакунарный ряд по обобщенным функциям Уолша. Коэффициенты $a_0, \ldots, a_{p^n-1}$ уравнения (1) вычисляются по заданным значениям p^n параметров $b_0, \ldots, b_{p^n-1}$ с помощью быстрого преобразования Виленкина – Крестенсона. Полученные результаты позволяют дать полное описание множества параметров $b_0, \ldots, b_{p^n-1}$, по которым определяются ортогональные вейвлеты $\psi_1, \ldots, \psi_{p-1}$ в $L^2(G)$, и для малых значений p и n оценить гладкость этих вейвлетов и масштабирующей функции φ. Кроме того, найдены условия, при которых финитное решение φ уравнения (1) стабильно в $L^2(G)$ и имеет линейно независимую систему "целочисленных"сдвигов. В случае Хаара изучаемый метод построения ортогональных вейвлетов приводит к вейвлетам, полученым в [7] для группы $\mathbb{F}_p((t))$.

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p-adic Brownian motion over Q_p

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In this talk, we consider p-adic Brownian motion over Q_p, which was first introduced by Bikulov [Theoret. and Math. Phys. 119 (1999) 594-604]. We shall present a direct construction of p-adic white noise and p-adic Brownian motion over Q_p by means of the Paley-Wiener method, which was originally employed by Bikulov and Volovich [Izvent. Math. 61 (1997) 537-552] for construction of p-adic Brownian motion over Zp. We shall also introduce p-adic random walk over Q_p/Z_p and discuss its weak convergence to p-adic Brownian motion.

Fractal theoretic aspects of local field

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Recent development of analysis on fractal is marked by importing method of function space. For example, theory of capacity is tightly related to Sobolev space and occasionally it is comparable with the Hausdorff measure.

Kaimanovich, Kumagai, Fukushima and Uemura focused on Besov space and invented methods to reveal analytic features of fractal sets. In particular, Besov space $B_{\alpha}^{2,2}(F)$ on fractal set F has a probabilistic significance, because it is regarded as the Dirichlet space associated with α-stable process on F. As for another importance in the Besov space, we recall that Jonsson and Wallin's trace theorem combined with Adam's imbeddding theorem provides us with Sobolev inequality on d-set in Euclidean space. This covers a theory of function spaces on fractal sets.

Investigations of stochastic process are made for example by Albeverio, Evans, Karwowski, Kochubei, Yasuda and Zhao. On the other hand, the field of p-adic numbers is contained as a d-set in larger field. Therefore, all this enables us to take potential theoretic approaches to local field based on stochastic process such as α-stable process.

The purpose of this talk is establishing the Sobolev inequality on d-set in local field. For the purpose, Jonsson and Wallin's trace theorem for potential space will be required.

As a related topic, we will focus also on a reasonable counterpart of Van der Corpt sequence on the field of p-adic numbers to shed light on another fractal theoretic aspect of the field.

Diffusion with ultrametric jumps

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Introduction in p-adic modeling of cognitive phenomena

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The aim of this talk is to provide a short introduction in p-adic modeling of cognitive phenomena. We start with introduction of p-adic mental space, its neurophysiological basis, modeling of psychological processes, including Freud's psychoanalysis.

On p-adic Gibbs measures of countable state Potts model on the Cayley tree

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We develop p-adic probability theory approaches to study of countable state nearest-neighbor Potts models on a Cayley tree over p-adic filed. Especially, we are interested in construction of p-adic Gibbs measure for the mentioned model. Such measures are more natural concrete examples of p-adic Markov processes (see [2], for definitions). When states are finite, say q, then the corresponding p-adic q-state Potts models on the same tree have been studied in [3,4]. There it was established that a phase transition occurs if q is divisible by p. This shows that the transition depends on the number of spins q. The case under consideration is different from the previous investigations, here spins take their values in a countable set.

For the model we give a construction of p-adic Gibbs measures which depends on weight λ. Using Kolmogorov extension Theorem [2], an investigation of such measures is reduced to examination of an infinite-dimensional recursion equation. Note that comparing with a real case, in a p-adic setting, à priori the existence of such kind of measures for the model is not known. Since, there is no much information about topological properties of the set of all p-adic measures defined even on compact spaces. In the real case, there is so called the Dobrushin's Theorem [1] which gives a sufficient condition for the existence of the Gibbs measure for a large class of Hamiltonians. Studying the derived equation under some condition on weights, we prove absence of the phase transition. Note that, for the real counterparts of the model, analogous results are unknown. It turns out that the found condition does not depend on values of the prime p, therefore, a similar fact is not true when the number of spins is finite. For the homogeneous p-adic Potts model, and show under that founded condition, we prove the existence of the p-adic Gibbs measure μ_λ. Then we establish boundedness one, and prove continuous dependence the measure μ_λ on λ. As well as we prove one limit theorem for μ_λ.

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p-Adic wavelets and their application to linear and nonlinear pseudo-differential evolutionary equations

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We introduce a new class of non-Haar multidimensional p-adic compactly supported wavelets [3]. In one-dimensional case this class includes the Kozyrev p-adic wavelets [4]. These wavelets form an orthonormal basis in $L^2(\mathbf{Q}_p^n)$. We derive a criterion for a multidimensional p-adic wavelet to be an eigenfunction for a pseudo-differential operator introduced in [1]. This criterion holds for a multidimensional fractional operator, i.e., these wavelets are eigenfunctions of the fractional operator. p-Adic wavelet bases are used to construct solutions of linear and nonlinear pseudo-differential evolutionary equations (see [2]).

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A non-Archimedean wave equation

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Let K be a non-Archimedean local field with the normalized absolute value $|\cdot|$. It is shown that a "plane wave" $f(t+\omega_1 x_1+\cdots+\omega_n x_n)$, where f is a Bruhat-Schwartz complex-valued test function on K, $(t,x_1,\ldots,x_n)\in K^{n+1}$, $\max\limits_{1\le j\le n}|\omega_j|=1$, satisfies, for any f, a certain homogeneous pseudo-differential equation, an analog of the classical wave equation. A theory of the Cauchy problem for this equation is developed.

p–Adic model of the genetic code

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We introduce the simple parametrization for the space of codons (triples of nucleotides) by 8×8 table. This table (which we call the diadic plane) possesses the natural 2-adic ultrametric. We show that after this parametrization the genetic code will be a locally constant map of the simple form. The local constancy of this map will describe degeneracy of the genetic code.

The map of the genetic code defines 2-adic ultrametric on the space of amino acids. We show that hydrophobic amino acids will be clustered in two balls with respect to this ultrametric. Therefore the introduced parametrization of space of codons exhibits the hidden regularity of the genetic code.

The fast Fourier–Heisenberg transform on the p-adic Heisenberg group as a natural quantum Fourier transform with the multiresolution properties of wavelet transforms, and an interface between the classical and quantum worlds

Valery Labunets

Быстрое преобразование Фурье-Гейзенберга на p-адической группе Гейзенберга в качестве истинного квантового преобразования Фурье с мультимасштабными свойствами вейвлет-преобразований, а также как интерфейс между классическим и квантовыми мирами

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Как известно, операторно-значное преобразование Фурье реализует процедуру квантизации Вейля. Она отображает классический мир на квантовый. Известно также, что набор операторно-значных преобразований Фурье, параметризованный различными постоянными Планка, формирует полный набор неприводимых представлений группы Гейзенберга над полем вещественных чисел. Само преобразование Фурье-Гейзенберга реализует набор Вейлевских процедур квантизации (параметризованный постоянной Планка), которые отображают классический мир на квантовые миры с различными постоянными Планка.

В работе доказывается, что имеет место иная ситуация, если рассматривать группу Гейзенберга над кольцами вычетов по модулю p^n и над p-адическими числами. Эта группа описывает фазовое пространство квантовых систем с конечным числом состояний, например, квантовые регистры. В этом случае группа Гейзенберга имеет неприводимые представления различных размерностей, а преобразование Фурье-Гейзенберга реализует набор процедур квантизации Вейля, которые отображают классический мир на набор неизоморфных квантовых миров различной размерности. Поэтому преобразование Фурье-Гейзенберга может трактоваться по-разному. Во-первых, это интерфейс между классическим и квантовыми мирами, во-вторых, это истинно квантовое преобразование Фурье, в том смысле, что оно ставит в соответствие классическому миру не его спектральное представление, а набор квантовых миров. В-третьих, так как квантовые миры различной размерности образуют башню (“матрешку”) уменьшающихся квантовых миров, то это преобразование можно рассматривать как вейвлет-преобразование, реализующее многомасштабные представлени квантовых миров.

В докладе строится процедура быстрого преобразовани Фурье-Гейзенберга. Приводятся блок-схемы быстрого алгоритма на классическом и квантовом компьютерах, т.е. даются классическая и квантовая реализации истинно квантового преобразования Фурье (в облике преобразования Фурье-Гейзенберга).

Real and p-adic fractal strings and their complex dimensions

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We will give an overview of the theory of complex dimensions of fractal strings, as developed in the two research monographs by Machiel van Frankenhuijsen and the author.

Kolmogorov widths in non-Archimedean spaces

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Since A. Kolmogorov [1] introduced the notion of width, various widths were widely investigated (see [2] for a survey). However, the obtained results concern linear metric spaces over the real numbers field or the complex numbers field. Apparently, for the first time Kolmogorov widths in linear metric space over a non-Archimedean field were considered by the author [3]. Some general results were obtained, and these results were applied to the class of function in the field of p-adic field, that satisfy Lipschitz condition of the degree 1. In this talk, we continue investigation of width problem for non-Archimedean spaces. Kolmogorov widths for the class of function with values in arbitrary complete non-Archimedean discretely normed field that have prescribed continuity module are found.

We use some notions from approximation theory. Details can be found, for example, in [2]. Let (X, d) be a metric space. Suppose $A \subset X$, $x \in X$. Denote

$$d(x, A) = \inf_{y \in A} d(x, y).$$

We say that $d(x, A)$ is a *distance* from x to A. Let K be bounded. Define *deviation* K from A as

$$\delta(K, A) = \sup_{x \in K} d(x, A). \tag{5}$$

The problem of finding $\delta(K, A)$ is known as the problem of *the best approximation* of K by A. Let $\mathcal{A}$ be some class of subsets of X. The value

$$d_{\mathcal{A}}(K) = \inf_{A \in \mathcal{A}} \delta(K, A) \tag{6}$$

is said to be $\mathcal{A}$-*width* of K.

Suppose E is a Banach space over a normed field $(\mathbb{K}, |\cdot|)$, the distance in E is induced by the norm, and $\mathcal{L}_n$ is the class of all n-dimensional vector subspaces of E. Then $d_n(K) \triangleq d_{\mathcal{L}_n}(K)$ is called *Kolmogorov* or n-*dimensional Kolmogorov* width.

Suppose (X, d) is an infinite compact ultrametric space [4]. Denote $D = \big\{d(x, y) \mid x, y \in X,\ x \neq y\big\}$. For a positive sequence $\alpha = (\alpha_0, \alpha_1, \ldots)$ we write $\alpha_i \downarrow 0$ if this sequence strictly decreases and tends to 0 as $i \to \infty$. The following result was announced in [5] and proved in [6].

Theorem 1 *$D = \{\alpha_1, \alpha_2, \ldots\}$, where $\alpha_i \downarrow 0$. In other words, in this case 0 is a unique limit point of D.*

We say that some metric space is *homogeneous* if every two closed balls of the same radius are isometric. Further we suppose that the space (X, d) is homogeneous. It is known [4] that for any $r > 0$ there exists a unique partition of X into disjoint balls of radius r. Denote ν_i the power of such partition for $r = \alpha_i$.

Suppose $(\mathbb{K}, |\cdot|)$ is a complete non-Archimedean discretely normed field. Denote $C(X, \mathbb{K})$ the non-archimedean Banach space of all continuous functions from X to $\mathbb{K}$ equipped with the supremum norm and V the range of the valuation $|\cdot|$. Let $w : \mathbb{R}^+ \to \mathbb{R}^+$ be a continuous strictly increasing function such that $w(0) = 0$. Let

$$H = \Big\{u \in C(X, \mathbb{K}) \,\Big|\, |u(x) - u(y)| \leq w(d(x, y)) \ \forall x, y \in X\Big\}.$$

One can show that

$$H = \Big\{u \in C(X, \mathbb{K}) \,\Big|\, |u(x) - u(y)| \leq w(\alpha_i), \text{ if } d(x, y) = \alpha_i\Big\}.$$

Define a function $\widetilde{w}$ on D by

$$\widetilde{w}(\alpha_i) = \max \Big[a \,\Big|\, a \in V, a \leq w(\alpha_i)\Big].$$

Theorem 2 *For any i*

$$d_{\nu_i}(H) = d_{\nu_i + 1}(H) = \ldots = d_{\nu_{i+1} - 1}(H) = \widetilde{w}(\alpha_i).$$

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Levy process in p-adics and hierarchical systems

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We consider the model proposed by Ogielski and Stein to describe relaxation in hierarchical systems.For our purpose we specify random walk on p-adic integer numbers,which occurs to be Levy process.A trajectory of a Levy path have some fractal dimension, which correspond to a fractional derivative of a pseudodifferential diffusion equation.

Probabilistic structures and probabilistic sets

V.M. Maximov

Вероятностные структуры и вероятностные множества

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Вводится понятие вероятностной структуры и вероятностного множества. Это суть алгебраические структуры, с помощью которых можно моделировать меры и случайные величины. Вероятностные структуры естественно строятся на булеане, аналогично тому как естественно возникают понятия решетки и булевой алгебры на булеане.

Доказывается теорема:

Всякая конечная вероятностная структура изоморфна вероятностной структуре булеана.

Относительно вероятностного множества предложена гипотеза, что всякое компактное вероятностное множество изоморфно отрезку вещественной оси [0,1], рассматриваемому с естественными операциями и топологией.

Algorithm of computation of vertex parts of p-adic Feynman integrals

Moukadas Missarov

Алгоритм вычисления вершинных частей p-адических фейнмановских интегралов

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Рассматриваются обобщенные p-адические фейнмановские амплитуды в координатном представлении, пропагаторы которых задаются как произвольная степень p-адической нормы аргумента. Предложен алгоритм вычисления вершинных частей амплитуд в схемах аналитической и размерной перенормировок, основанный на разложениях p-адических интегралов по иерархическим семействам.

From Data to the p-Adic or Ultrametric Model: Recent Results

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p-Adic or ultrametric modeling of data, representing phenomena or processes, can advance beyond the current state of the art in data analysis and data mining. We consider a number of areas to illustrate the new perspectives opened up by such analysis. We show how very high dimensional data are best considered as emedded in an ultrametric space. In a range of examples, we show how high dimensional spaces endowed with a scalar product can be embedded in an ultrametric topology. As a case study, we consider the segmentation of very high frequency financial signals. We look at the role played by symmetry in data analysis and data mining, and explain why hierarchy provides a unifying view. As pointed out by Benois-Pineau and Khrennikov (2007), a p-adic viewpoint is very powerful for change detection in data streams. We look at the application of ultrametric data analsis for the understanding of change, anomaly and innovation. We use film scripts, which constitute a convenient way of representing narrative, and indeed the flow of thought and language (Chafe, 1979).

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Beta-function of Bruhat–Tits building and p-adic Berezin kernels

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For the space Lat_n of all the lattices in a p-adic n-dimensional linear space we obtain an analog of matrix beta-functions; this beta-function has a degeneration to the Tamagawa zeta-function. We propose an analog of Berezin kernels for Lat_n. We obtain conditions of positive definiteness of these kernels and explicit Plancherel formula.

Approximating 2-adic polynomial dynamical systems

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Geometrization of quantum physics

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The new concept of quantum physics is suggested where all matter (particles, fields, quanta, atoms) is specific deformations of the space itself—the space topological defects. This explains the light velocity invariance and all peculiar properties of quantum formalism. This gives also possibility for overcoming difficulties of the many-body problem. Preliminary results: Journ.of Phys.:Conf.Ser., 2007, 67, 012037 (arXiv:0706.3461). The work was presented and discussed at the conference in Vaxjo, Sweden

Compactification of $\mathbb{Z}$ via periodicity

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The aim of the present talk is to show very close relations between arithmetics and the theory of commutative Banach algebras. Our approach is based on the compactification of $\mathbb{Z}$ via *periodicity*, which seems very natural. Indeed, on the one hand, the sum (or the product) of m– and n–periodic functions on $\mathbb{Z}$ has each common multiple of m and n as period. On the other hand, if a function f is m– and n– periodic simultaneously then any common divisor of m and n is also a period of f. As divisibility is a main notion of arithmetics, the algebra $\mathbb{A}$ of all complex–valued periodic functions on $\mathbb{Z}$ (with termwise operations) must play an important role in number theory. In fact, there are many works (cf. [2,4,5]) concerned with the *almost–periodic* functions which are limit elements of $\mathbb{A}$ with respect to various norms. We call by $\mathbb{A}$*–character* any non–zero multiplicative linear functional $\psi : \mathbb{A} \to \mathbb{C}$. Denote by δ^n the character $u \mapsto u(n)$. The set $\mathbb{G}$ of all characters is a commutative ring with the additive $\oplus$ and multiplicative $\odot$ convolutions as ring operations, zero δ^0 and the unit δ^1. Moreover, $\delta^m \oplus \delta^n = \delta^{m+n}$, $\delta^m \odot \delta^n = \delta^{mn}$. By associating each integer m with the element δ^m of the ring $\mathbb{G}$, we obtain the *canonical embedding* $\mathbb{Z} \to \mathbb{G}$ which is a strict homomorphism of the rings.

We call by *p–equivalent* two characters φ and ψ coinciding on the finite–dimensional subalgebra of all p–periodic functions. Let $V_p(\psi)$ be the p–equivalence class (*p–cluster*) containing the character ψ. These classes form a base of neighborhoods of the point $\psi \in \mathbb{G}$, which defines a topology τ compatible with the ring structure of $\mathbb{G}$. The ring $(\mathbb{G}, \tau)$ is a compact Hausdorff space including the image of $\mathbb{Z}$ under canonical embedding as dense subset. A homeomorphic image of $(\mathbb{G}, \tau)$ can be realized as a discontinuum $\mathbb{D}$ which consists of final values of increasing bounded trajectories of some $\mathbb{R}$–valued branching stochastic process. Besides, the Lebesgue measure $\mathrm{mes}(E)$ of each measurable subset $E \subseteq \mathbb{D}$ is equal to the Haar measure of its preimage $F \subseteq \mathbb{G}$.

The main result of the paper [6] asserts the isomorphism of the classical polyadic ring with divisibility topology (Van Dantzig–Novoselov's model [1,3]) and the topological ring $(\mathbb{G}, \tau)$ of $\mathbb{A}$–characters with convolution ring operations and the cluster topology.

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p-Adic S.N. Bernstein inequality

Ya. V. Radyno

p-Адическое неравенство С.Н. Бернштейна

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Тригонометрические полиномы порядка меньше либо равного ν являются частными случаями целых функций экспоненциального типа меньше либо равного ν. Напомним, что целая функци $\varphi()$, где $z = x+iy$, называется функцией экспоненциального типа меньше либо равного ν, если справедливо неравенство

$$|\varphi(z)| \leq Ce^{|\nu|z} \quad \text{для всех } z \in \mathbf{C}.$$

Обозначим через $A_{\nu,q}(\mathbf{R})$, $1 \leq q \leq +\infty$, множество всех целых функций экспоненциального типа меньше либо равного ν, которые, как функции от действительного переменного $x \in \mathbf{R}$, принадлежат пространству $L_q(\mathbf{R})$. Например, функция $\frac{\sin \nu z}{z}$ принадлежит пространству $A_{\nu,q}(\mathbf{R})$, $1 < q \leq +\infty$. Для функций из $\varphi \in A_{\nu,q}(\mathbf{R})$ справедливо неравенство Бернштейна [1, 2]:

$$||\varphi'||_{L_q(\mathbf{R})} \leq \nu ||\varphi||_{L_q(\mathbf{R})}.$$

Неравенство точно, так как существуют функции для которых в нем достигается знак равенства. Более того, это неравенство характеризует пространство $A_{\nu,q}(\mathbf{R})$, а именно, если для функции $\varphi \in L_q(\mathbf{R})$ справедливо неравенство

$$\sup_{n \geq 0} \frac{||\varphi^{(n)}||_{L_q(\mathbf{R})}}{\nu^n} < +\infty,$$

то $\varphi \in A_{\nu,q}(\mathbf{R})$ [1].

Мы интересуемся характеристикой пространства $A_{\nu,q}(\mathbf{R})$, где вместо оператора дифференцировани стоит псевдодифференциальный оператор Владимирова D [3].

Справедлива следующа

Теорема. *Замкнутое подпространство функций $u \in L_2(\mathbf{Q}_p)$, для которых справедливо неравенство Бернштейна*

$$||Du||_{L_2(\mathbf{Q}_p)} \leq p^\nu ||u||_{L_2(\mathbf{Q}_p)}$$

состоит из локально постоянных функций $u \in L_2(\mathbf{Q}_p)$, параметр постоянства которых $\geq -\nu$.

Другими словами, p-адической моделью функций экспоненциального типа $\leq p^\nu$ являются локально постоянные функции из $L_2(\mathbf{Q}_p)$ с параметром постоянства $\geq -\nu$.

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P-adic Splines and Riesz-Volkenborn's Potential

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p-Adic linear splines were introduced in [1] and used for approximation of real-valued functions, but for p-adic valued functions they were not suitable. p-Adic splines introduced in [2] solve this problem and open new fact: p-adic spline is an integral sum of Volkenborn's integral from a density function multiplied by Riesz kernel. Such integral we call Riesz-Volkenborn's potential.

For any prime number p denote by $\mathbb{Q}_p$ the set of p-adic numbers equipped by the norm $|x|_p$. Denote by $B_\gamma[a] = \{x \in \mathbb{Q}_p \ : \ |x-a|_p \le p^\gamma\}$, and by $I_{B_\gamma[a]}$ the characteristic function of the ball. Let $\alpha \in \mathbb{Q}$, $\alpha > 1$. Denote by $\mathbb{K}$ the minimal field containing $\mathbb{Q}_p$ and $\{p^\alpha\}$. The norm $|\cdot|_p$ is prolonged to $\mathbb{K}$ by using standard properties of $|\cdot|_p$: $|p^\alpha|_p = p^{-\alpha}$, $|ab|_p = |a|_p|b|_p$, $|a+b|_p \le \max(|a|_p, |b|_p)$.

Recall that Volkenborn's integral of $f \in C^1(\mathbb{Z}_p;\mathbb{K})$ is a number $\int\limits_{\mathbb{Z}_p} f(x)dx := \lim\limits_{n\to\infty} p^{-n} \sum\limits_{j=0}^{p^n-1} f(j)$, see [3] for details.

It is a linear continuous functional on $C^{(1)}(\mathbb{Z}_p;\mathbb{K})$. The next inequality holds true

$$\left| \int_{\mathbb{Z}_p} f(x)dx \right|_p \le p\|f\|_1, \text{ where} \tag{7}$$

$$\|f\|_1 = \max\left(\max_{x\in\mathbb{Z}_p} |f(x)|_p, \max_{x\in\mathbb{Z}_p} |f'(x)|_p \right) .$$

We fix $\alpha > 1$ and consider the function $\mathbb{Z}_p \ni x \mapsto |x|_p^{-\alpha} \in \mathbb{K}$.

Lemma 1. *The function $\varphi(x) = |x|_p^{-\alpha}$ is a continuously differential and $\varphi'(x) \equiv 0$.*

Theorem 1.[4] *For any $a \in \mathbb{Z}_p$, $m \in \mathbb{N}\cup\{0\}$, and $\alpha \in \mathbb{Q}$, $\alpha > 1$ there exist p-adic splines $L_n(x,\alpha)$ such that $L_n(x,\alpha) \rightrightarrows I_{B_{-m}[a]}(x)$ as $n \to \infty$.*

Theorem 2. *For the sequence $\{L_n(x,\alpha)\}_{n=1}^\infty$ from Theorem 1 there exists a function $\Lambda^{\alpha,m}$ such that $L_n(x,\alpha) \rightrightarrows \int\limits_{\mathbb{Z}_p} \frac{\Lambda^{\alpha,m}(t)dt}{|t-x|_p^\alpha}$ as $n \to \infty$.*

From theorems we immediately have $I_{B_{-m}[a]}(x) = \int\limits_{\mathbb{Z}_p} \frac{\Lambda^{\alpha,m}(t)dt}{|t-x|_p^\alpha}$. Since the linear span of characteristic functions of all balls on $\mathbb{Z}_p$ is dense in $C(\mathbb{Z}_p;\mathbb{K})$ then by linearity and continuity of Volkenborn's integral we shall have the integral representation for any function from $C(\mathbb{Z}_p;\mathbb{K})$.

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Fourier transform of vector-valued functions over p-adic field

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In paper [1] Polish mathematician Kwapień considered Fourier transform of functions taking value in Banach spaces. One of his results is up to notation the following. Let X be a Banach space. Consider space $L_2(\mathbf{R}, X)$ of functions on the real axis taking value in X, and square-integrable in Bochner sense. Banach space X is isomorphic to a Hilbert space if and only if Fourier transform (which is initially defined on a suitable dense subset of $L_2(\mathbf{R}, X)$)

$$\mathcal{F} : L_2(\mathbf{R}, X) \to L_2(\mathbf{R}, X) : x(t) \mapsto (\mathcal{F}x)(s) = \int_{\mathbf{R}} e^{-2\pi ist} x(t)dt.$$

is a bounded operator.

Now we are interested in Fourier transform of X-valued functions on $\mathbf{Q}_p$. Space $L_2(\mathbf{Q}_p, X)$ of Bochner square-integrable functions contains a dense subset $L_2(\mathbf{Q}_p) \otimes X$ where Fourier transform act by

$$\mathcal{F} : \sum_{k=1}^{N} \varphi_k(t) \cdot \vec{x}_k = \sum_{k=1}^{N} (\mathcal{F}\varphi_k)(s) \cdot \vec{x}_k.$$

We prove the following

Theorem *Banach space X is isomorphic to a Hilbert space if and only if Fourier transform*

$$\mathcal{F} : S(\mathbf{Q_p}, X) \to L_2(\mathbf{Q_p}, X) : x(t) \mapsto (\mathcal{F}x)(s) = \int_{\mathbf{Q_p}} e^{2\pi i\{st\}_p} x(t)dt.$$

is a bounded operator.

Proof uses a smaller subspace of locally constant compactly-supported X-valued functions $\mathcal{S}(\mathbf{Q}_p, X) \simeq \mathcal{S}(\mathbf{Q}_p) \otimes X \subset L_2(\mathbf{Q}_p) \otimes X$.

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Non-Commutative Geometry and Quantization of the Universal Teichmüller Space

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The universal Teichmüller space $\mathcal{T}$ consists of quasisymmetric homeomorphisms of the circle S^1 (i.e. orientation-preserving homeomorphisms of S^1, extending to quasiconformal maps of the disc), normalized modulo Möbius transformations. It has a natural Kähler structure and contains all classical Teichmüller spaces (of compact Riemann surfaces of finite genus) as complex submanifolds. Moreover, $\mathcal{T}$ includes also the space $\mathcal{S}$ of diffeomorphisms of the circle, normalized modulo Möbius transformations, which can be considered as a "smooth" part of $\mathcal{T}$. The space $\mathcal{S}$ can be quantized, using its embedding into an infinite-dimensional Siegel disc. However, this method does not apply to the whole universal Teichmüller space. For its quantization we use the "quantized calculus" of A.Connes.

On the dynamical systems with 2-adic time
(a talk joint with V. Dremov and P. Vytnova)

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A general concept of dynamical system with non-archimedean time will be suggested. It will be illustrated by a certain limit of the dynamics on the sets of 2^n-periodic points of real quadratic maps.

p-Adic refinable functions and MRA-based wavelets

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The notion of *p-adic multiresolution analysis* (*MRA*) is introduced and a general scheme for its construction is described. We study refinement equations whose solutions (refinable functions) are boundedly limited. A criterion of orthogonality is found for a class of such functions. A method for construction of refinable functions generating MRAs is given. In particular, this scheme lids us to the Haar "natural" refinement equation whose solution is the characteristic function of the unit disc. This equation reflects the fact that the characteristic function of the unit disc is a sum of p characteristic functions of mutually disjoint discs of radius p^{-1}. This "natural" refinement equation was introduced in [1]. In contrast to the real setting, the refinable function generating p-adic Haar MRA is 1-periodic, which never holds for real refinable functions. This fact implies that there exist *infinity many* different orthonormal wavelet bases generated by the same Haar MRA. For $p = 2$ all such bases are described in [3]. One of these bases coincides with Kozyrev's basis [2].

A criterion for multidimensional p-adic wavelets to be eigenfunctions for a pseudo-differential operator is derived.

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Critical exponents in hierarchical models

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We consider 4-component fermionic Dyson model on a hierarchical lattice. Critical exponent ν was calculated without using a perturbative technique. We show that $\nu = \ln p / \ln x$, where p is a parameter of lattice, x is a root of the polynomial equation

$$x^2(p^{-1} - 1) + x\left(4a + 6 + 4a^{-1} + 2p^{-1}\right) + \\ + p^{-1} + p - 4a^2 - 8a - 10 - 8a^{-1} - 4a^{-2} + \\ + x^{-1}\left(4a + 6 + 4a^{-1} + 2p\right) + (p-1)x^{-2} = 0,$$

where $a = p^{\varepsilon + 1/2}$, $\varepsilon = \alpha - 3/2$, α is a parameter of renormalization group.

This result was compared to a known result for ν calculated up to 3rd order in powers of ε in p-adic φ^4-model with $O(N)$-symmetry (for $N = -4$). Earlier it was obtained using Feynman diagrammatic technique. Calculations show that both results coincide. It means that both perturbation theory and rigorous methods developed for hierarchical and p-adic models give the same results.

For general N we show that critical exponent ν may be calculated via finding roots of some finite polynom. An interesting problem is to find analogous properties in Euclidean models.

Two-periodical Dynamics in Finite Extensions of the p-adic Number Field

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We investigate two-periodic points of a certain class of dynamical systems, defined over the field of p-adic numbers. We determine the topological properties of these points, and the nature of the smallest finite extension in which the periodic points reside.

p-adic Schrodinger-Type Operator with Point Interactions

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p-adic Schrodinger-Type Operator $D^a + V$ is studied, where D^a is the operator of fractional differentiation and V is a singulat potential containing Dirac delta function concentrated at the set of arbitrary n p-adic points. It is showed that such a problem is well-posed for $a > 1/2$ and the singular potential is form-bounded for $a > 1$. In the latter case, the spectral analysis of self-adjoint realisations is carried out.

Boundary-value problems for open and closed p-adic strings theory

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For description the tachyon dynamics for p-adic strings the nonlinear pseudodifferential equation

$$\psi^n(t) = (e^{x_0 \partial^2_{tt}} \psi)(t), \quad t \in \mathbb{R}, \tag{1}$$

has been suggested[1-2]. The boundary conditions are supplied: for open string $(x_0 = 1/4, n = p)$

$$\psi(-\infty) = -1, \psi(\infty) = 1, p - odd, \quad \psi(-\infty) = 0, \psi(\infty) = 1, p - even;$$

(p is any prime number); for closed strings $(x_0 = 1/2, n = p^2)$

$$\psi(-\infty) = \psi(\infty) = 1 \quad or \quad \psi(-\infty) = \psi(\infty) = 0.$$

We discuss the following topics concerning the b.-v.ps posed.
• Existens or non-existens and uniqueness of solutions[3-4].
• À priori properties and estimates of solutions[5].
• Hermite-series expansion of solutions[5].
• Reducing the posed b.-v.ps to nonlinear heat equation[5]:

$$u_x = u_{tt}, 0 < x \leq x_0, t \in \mathbb{R}, \quad u(0,t) = \psi(t), u(x_0,t) = \psi^n(t). \tag{2}$$

We point out that the variables x and t in equation (2) are interchanged compared with the classical heat equation.

• Tchebyshev-series expansion of periodic solutions to b.-v.p. (2).

• Applications of the Gauss quadrature formulas to approximations the equation (1).

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Quantization of the Riemann Zeta-Function and p-Adic Pseudodifferential Equations

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Motivated by the theory of p-adic strings we consider quantization of the Riemann zeta-function. We treat the Riemann zeta-function as a symbol of a pseudodifferential operator $\zeta(1/2 + i\Box)$ where $\Box$ is the d'Alembert operator and study the corresponding classical and quantum field theories. We show that the pseudodifferential equation for the zeta-function field is equivalent to a family of the Klein-Gordon equations with masses defined by the zeros of the Riemann zeta-function. Quantization of the L-functions in the Fermat-Wiles theory and in the Langlands program is also indicated.

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On unification of real and p-adic theories

E.I. Zelenov

Об объединении вещественной и p-адической теорий

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Начиная с работ В.С.Владимирова и И.В.Воловича по p-адической математической физике общепринятой является точка зрения, что результатом эксперимента (наблюдения) является рациональное число.

Предлагается несколько видоизмененная точка зрения. А именно, **результатом эксперимента всегда является конечна последовательность рациональных чисел,** поскольку эксперимент это всегда серия наблюдений.

Исходя из этого постулата, естественным выглядит выбор в качестве физического пространства P слабого (т.е. только конечное число сомножителей отлично от нуля) прямого произведения континуум экземпляров аддитивной группы поля рациональных чисел Q:

$$P = \prod\nolimits_{i\in I}^{*} Q^i,$$

где I - множество индексов мощности континуум. Заметим, что P является аддитивной абелевой группой.

Естественность выбора P в качестве физического пространства подтверждается следующим простым Предложением.

Предложение 1 *Группа P алгебраически изоморфна аддитивной группе $R^{(d)}$ вещественных чисел с дискретной топологией. Группа P алгебраически изоморфна аддитивной группе $Q_p^{(d)}$ поля p-адических чисел с дискретной топологией.*

Доказательство следует из того факта, что $R^{(d)}$ и $Q_p^{(d)}$ можно рассматривать как линейные пространства над полем Q рациональных чисел с несчетным базисом Гамеля. (Э.Хьюитт, К.Росс. Абстрактный гармонический анализ).

Основной вопрос заключается в следующем - возможно ли построение содержательной теории, если в качестве пространства физических величин (например, координат и импульсов) служит пространство P?

В качестве примера такой теории рассмотрим теорию представлений коммутационных соотношений одномерной квантовой механики.

Пусть $V = P \times P$ - фазовое пространство классической системы, $\beta(x, y), x \in V, y \in V$ невырожденный антисимметричный бихарактер на

$V \times V$, т.е. β есть функция из $V \times V$ в поле комплексных чисел C, $\beta : V \times V \to C$, така что $|\beta(x, y)| = 1, x \in V, y \in V$, $\beta(x_0, y)$ являетс характером V при любом фиксированном x_0, $\beta(x, y_0)$ является характером V при любом фиксированном y_0, $\beta(x, y) = \overline{\beta(y, x)}$, при этом из справедливости равенства $\beta(x_0, y) = 1$ для всех $y \in V$ следует $x_0 = 0$.

Представлением коммутационных соотношений будем называть пару (H, W), где H - комплексное Гильбертово пространство, а W - отображение из V в семейство унитарных операторов на H, удовлетворяющие соотношениям

$$W(x)W(y) = \beta(x, y)W(x + y).$$

Неприводимость и унитарная эквивалентность представлений коммутационных соотношений определяется стандартным образом.

Прежде чем сформулировать основные результаты, опишем вложение группы P в собственную группу характеров $\hat{P}$. Через Σ обозначим группу характеров аддитивной группы поля рациональных чисел, $\Sigma = \hat{Q}$. (Напомним, что это фактор-группа группы аделей по подгруппе главных аделей). Из определения P следует, что $\hat{P} = \prod_{i \in I} \Sigma^i$. Если в качестве модели P выбрать $R^{(d)}$, становится понятным, что множество элементов P находится во взаимнооднозначном соответствии с множеством непрерывных характеров R, а множество элементов $\hat{P}$ со множеством всех характеров R, таким образом определено вложение $P \subset \hat{P}$. Следовательно, определено вложение $V \subset \hat{V}$.

Замечание 1 *Поскольку группа непрерывных характеров на R изоморфна R, $R \simeq \hat{R}$, то компактификация Бора bR группы R совпадает с группой $\widehat{R^{(d)}}$, которая, как мы установили ранее, изоморфна группе $\prod_{i \in I} \Sigma^i$. Вложение $P \subset \hat{P}$ есть каноническое вложение группы P в компактификацию Бора bP этой группы.*

Теорема 1 *Множество классов сепарабельных неприводимых унитарно эквивалентных представлений коммутационных соотношений находится во взаимооднозначном соответствии с множеством элементов фактор-группы $\hat{V}/V$.*

Набросок доказательства. Воспользуемся обобщением Вейл теоремы Стоуна-фон Неймана о единственности с точностью до унитарной эквивалентности неприводимых представлений коммутационных соотношений. (А.Вейль. Об одной группе унитарных операторов). Согласно Вейлю, существует единственное с точностью до унитарной эквивалентности непрерывное неприводимое представление коммутационных соотношений над группой $P \times \hat{P}$. P вкладывается в компактную группу $\hat{P}$ как дискретная подгруппа. Ограничим представление коммутационных соотношений на подгруппу

$\{(0,x), x \in \hat{P}\}$ и воспользуемся теоремой о продолжении характера группы $P \subset \hat{P}$ на группу $\hat{P}$. Это можно сделать $\hat{P}/P$ различными способами. Отсюда можно получить утверждение Теоремы.

Связь вещественной и p-адической теорий дает следующая Теорема.

Теорема 2 *Пусть H - сепарабельное Гильбертово пространство. Для любого неприводимого представления (H, W) коммутационных соотношений над $V = P \times P$ и для любого простого p, включая $p = \infty$, существует характер $\chi_p, \chi_p \in \hat{V}$ группы V такой, что представление $(H, \tilde{W})$, где $\tilde{W}(x) = \chi_p(x)W(x), x \in V$ - непрерывное представление коммутационных соотношений над $Q_p \times Q_p, (R = Q_\infty)$.*

Доказательство является несущественной модификацией доказательства Дж. Славного аналогичного утверждения дл вещественного случая (Дж. Славный. О факторных представлениях C^*-алгебры канонических коммутационных соотношений).

Таким образом, мы получили следующий результат. Одно и то же представление коммутационных соотношений над физическим фазовым пространством может рассматриваться как непрерывное представление над вещественным фазовым пространством и как непрерывное представление над p-адическим фазовым пространством. Переход к соответствующему непрерывному представлению осуществляется путем умножения на характер группы V.